

Practical Model Predictive Control Structures for Non-Self Regulating (Integrating) Processes

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ABSTRACT

Model Predictive Control (MPC) refers to a family of control algorithms that employ an explicit model to predict the future behavior of the process. A simple and common form of MPC control is the Smith predictor. The Smith predictor has been known to enhance control of processes with large dead times. The use of the original Smith predictor control structure with integrating processes will result in an offset problem during certain disturbance rejection scenarios. In this work, a modified class of Smith predictor control structures are presented that overcome this problem.

Another popular MPC algorithm is Dynamic Matrix Control (DMC). One of the limitations of classical DMC is that it requires the internal DMC step response process model to describe a stable plant. To allow for the differences in the dynamics of integrating processes over stable processes, the DMC algorithm is modified to account for this unstable behavior.

Designing a Dynamic Matrix Controller (DMC) is challenging because of the number of tuning parameters that affect closed loop performance. The tuning parameters required to implement DMC include: the sample time; the prediction, model and control horizons; and the move suppression coefficient. The move suppression coefficient is used as the key tuning parameter to obtain desirable DMC performance. This work describes and demonstrates methods for calculating the complete set of DMC tuning parameters for integrating processes with long dead times.

INTRODUCTION

Self regulating processes seek a natural steady state operating level in open loop if the manipulated and disturbance variables are held constant for a sufficient period of time. It is not uncommon in manufacturing and production operations, however, for some temperature, level, pressure and other measured process variables to grow in an unbounded manner over time when perturbed in open loop by a manipulated or disturbance variable. Such non-self regulating, or integrating, processes are surprisingly challenging to control. If the integrating process also displays a large dead time, the control

challenge is compounded. An additional challenge is that the tuning correlations proven for self regulating processes can yield poor performance when applied to integrating processes.

A large time delay in the process response increases the difficulty for properly choosing and tuning a controller. Time delays occur in manufacturing and process operations, for example, due to the presence of transportation lags, recycle loops, and sampling and analysis. One approach is to detune the controller to prevent the closed loop response from becoming oscillatory or unstable.

An alternative solution is model predictive control (MPC), which can markedly improve closed loop performance in the presence of large time delay. MPC incorporates a dynamic process model as part of the architecture of the controller. For success in implementation, the model must reasonably describe the controller output to measured process variable dynamic behavior.

An important aspect of the block diagram construction lies in where the disturbances or loads enter the system. As shown in Fig. 1, a type-I disturbance describes a load that is applied to the system prior to entering the process block. A type-II disturbance describes a load that is applied after leaving the process block.

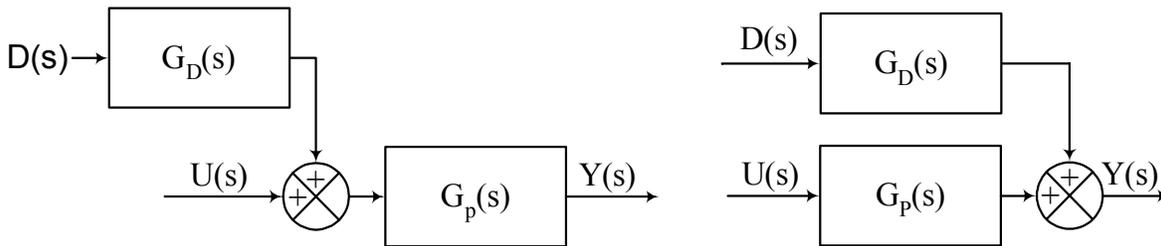


FIG. 1 - TYPE-I (LEFT) AND TYPE-II (RIGHT) DISTURBANCE

The design and tuning of controllers for integrating processes follows a simple three step procedure: collect dynamic process data, fit the data with a simple linear model, and use the model parameters in correlations to obtain tuning parameter values. This procedure is detailed and demonstrated in the remainder of this paper.

THE SMITH PREDICTOR

The Smith predictor is the most common MPC formulation for dead time compensation. Figure 2 shows the block diagram of the original Smith predictor with a type-I disturbance [1, 2-8]. In Fig. 1, $G_p(s)$ represents the actual process and $G_0(s)$ represents the ideal (with no dead time) model of the process.

The short-comings of the original Smith predictor control structure can be demonstrated even in the ideal case. Define:

$$G_p(s) = \frac{K_p^*}{s} e^{-\theta_p s} \tag{1}$$

$$G_0(s) = \frac{K_p^*}{s} ; G_T(s) = e^{-\theta_p s} \tag{2}$$

$$G_C(s) = K_C \left(1 + \frac{1}{\tau_I s} \right) \tag{3}$$

The closed-loop relationship of Fig. 2 can be derived:

$$Y(s) = \frac{K_c \left(1 + \frac{1}{\tau_I s}\right) \frac{K_p^*}{s} e^{-\theta_p s}}{1 + K_c \left(1 + \frac{1}{\tau_I s}\right) \frac{K_p^*}{s}} Y_{sp}(s) + \frac{K_p^*}{s} e^{-\theta_p s} \left(1 - \frac{K_c \left(1 + \frac{1}{\tau_I s}\right) \frac{K_p^*}{s} e^{-\theta_p s}}{1 + K_c \left(1 + \frac{1}{\tau_I s}\right) \frac{K_p^*}{s}}\right) D(s) \quad (4)$$

Assuming there is no change in setpoint, $Y_{sp}(s)=0$, but there is some type of disturbance, for example, $D(s)=1$ (unit disturbance), then at time equal to infinity, $Y(s)/D(s)$ is derived via l'Hopital's rule [4] as:

$$\lim_{s \rightarrow 0} \left(\frac{Y(s)}{D(s)} \right) = \lim_{s \rightarrow 0} \left[\frac{K_p^*}{s} e^{-\theta_p s} \left(1 - \frac{K_c \left(1 + \frac{1}{\tau_I s}\right) \frac{K_p^*}{s} e^{-\theta_p s}}{1 + K_c \left(1 + \frac{1}{\tau_I s}\right) \frac{K_p^*}{s}}\right) \right] = K_p^* \theta_p \quad (5)$$

As shown in Eq. 5, a disturbance to the process yields a steady-state offset in the system equal to the product of the integrator gain and dead time of the process. It should be noted that this unfavorable characteristic of the original Smith predictor occurs only in the presence of a type-I disturbance.

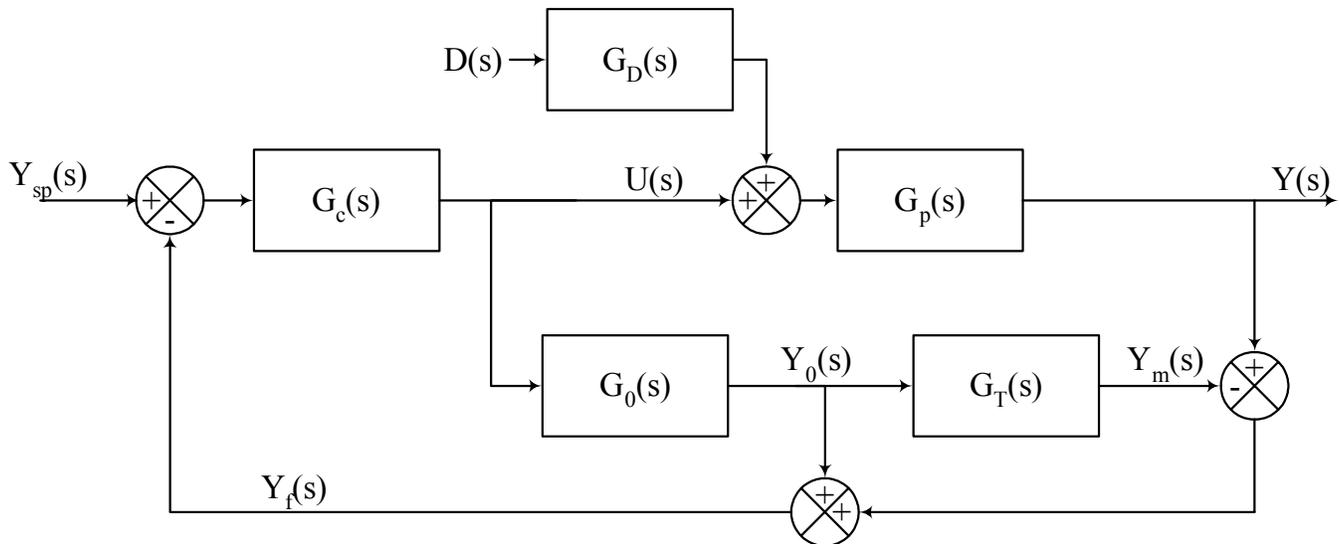


FIG. 2 - THE ORIGINAL SMITH PREDICTOR ARCHITECTURE

To avoid this offset problem, several authors [2, 5-7] have suggested similar modifications to the original Smith predictor design as shown in Fig. 3. This consists of an additional feedback path from the difference of the plant output $Y(s)$ and the model output $Y_m(s)$ to the controller input $U(s)$.

The transfer function $F(s)$ in Fig. 3 is chosen to eliminate the steady-state offset caused by the disturbance. It has been shown [5, 6] that the signal leaving the $F(s)$ transfer block estimates the input load $D(s)G_D(s)$ and enables load disturbance rejection. $F(s)$ also decouples the set point tracking response from the disturbance rejection response. In this work, the $F(s)$ will be determined by the method detailed in the earlier work of Mataušek in [5, 6]. In that work, Mataušek specified $F(s)$ as:

$$F(s) = \frac{1}{2\theta_p K_p^*} \quad (6)$$

Note that the ability of the Smith predictor to reduce the influence of dead time on controller performance is directly related to how well the model describes the actual process dynamics.

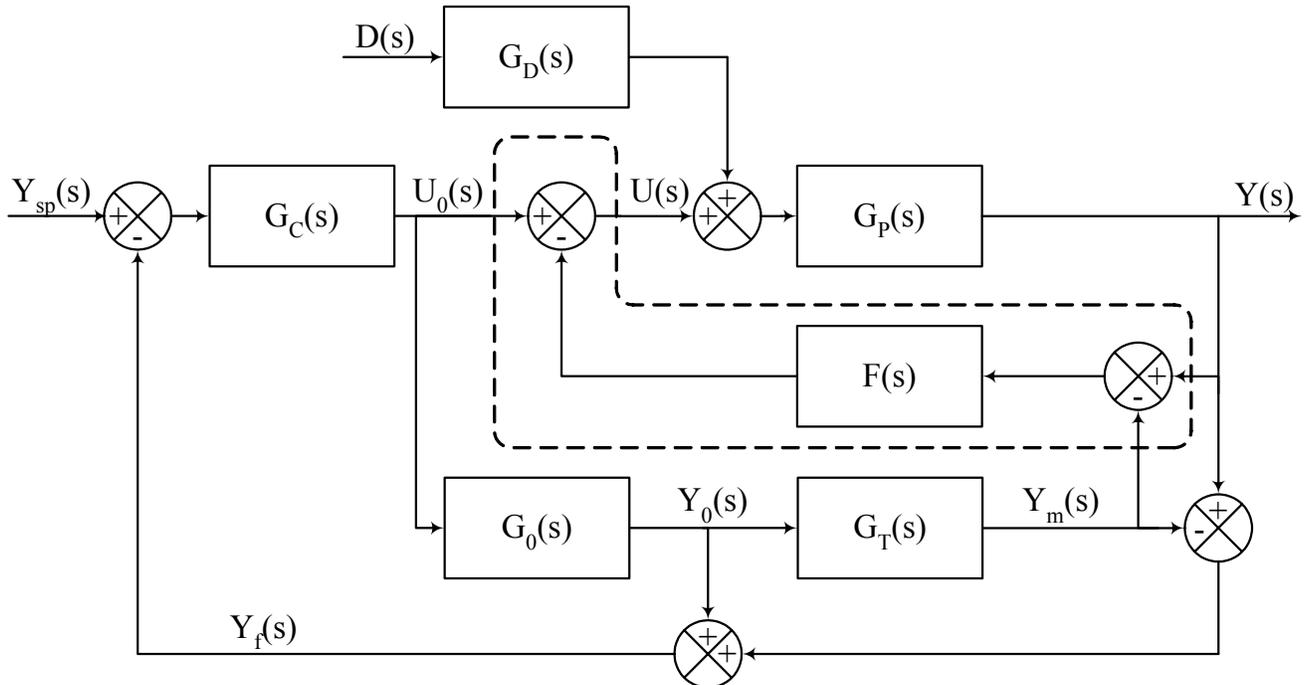


FIG. 3 - THE MODIFIED SMITH PREDICTOR ARCHITECTURE

DYNAMIC MATRIX CONTROL

Dynamic Matrix Control (DMC) is the most popular MPC algorithm used in manufacturing and production operations [10, 16, 23]. A major part of DMC's appeal stems from the use of a step response model of the process and a simple quadratic performance objective function. The objective function is minimized over a prediction horizon to compute the optimal manipulated variable moves as a least squares problem.

Modification of the DMC Algorithm

One of the limitations of classical DMC is that it requires the internal DMC step response process model to describe a stable plant [11, 14]. The predicted process variable profile, $\hat{y}(n+j)$, over j sampling instants ahead of the current time, n , is calculated as:

$$y(n+j) = y_0 + \underbrace{\sum_{i=1}^j a_i \Delta u(n+j-i)}_{\text{Effect of current and future moves}} + \underbrace{\sum_{i=j+1}^{N-1} a_i \Delta u(n+j-i)}_{\text{Effect of past moves}} \quad (7)$$

where a_i is the i^{th} unit step response coefficient of the process.

A modified formulation of DMC [16, 23] for integrating processes uses the fact that the measured process variable response due to the past controller output moves is just a straight line passing through the current measurement of the process variable at the current control instant. Hence, the predicted process variable profile is calculated as a slope projection. The modified DMC algorithm calculates the predicted process variable profile, $\hat{y}(n+j)$ as:

$$\hat{y}(n+j) = y_o + j[y(n) - y(n-1)] \quad (j = 1, 2, \dots, N) \quad (8)$$

where $y(n)$ is the process variable measurement at the current control instant and $y(n-1)$ is the process variable measurement at the previous control instant. Similarly, the disturbance estimate, $d(n+j)$, is modified as:

$$d(n+j) = d(n) = y(n) - y_o - [y(n) - y(n-1)] \quad (9)$$

DMC Tuning Rules for Integrating Processes:

The unusual dynamics of an integrating process require the use of a First Order plus Dead Time (FOPDT) integrating model approximation. A FOPDT Integrating model approximation has the form:

$$\frac{dy(t)}{dt} = K_p^* u(t - \theta_p) \quad \text{or} \quad \frac{Y(s)}{U(s)} = \frac{K_p^* e^{-\theta_p s}}{s} \quad (10)$$

where K_p^* is the integrator gain and θ_p is the effective dead time. Past researchers [17, 18, 24] have used a FOPDT Integrating model for tuning PI and PID controllers. It is emphasized here that the use of this FOPDT Integrating model approximation is employed only in the derivation of the tuning parameters used in DMC. The examples presented later in this work employ a step response model formed using actual process data upon implementation.

Using the FOPDT Integrating model parameters, the sample time, T , is computed such that the process is sampled two to three times per effective dead time [1, 23] or:

$$T \leq 0.5\theta_p \quad (11)$$

This value of sample time balances the desire for a low computation load (a large T) with the need to properly track the dynamic behavior of an integrating process (a small T). Recognizing that many control computers restrict the choice of T [10, 11], the remaining tuning rules permit values of T other than that compute by Eq. 12 to be used.

The discrete dead time is calculated in integer samples as:

$$k = \text{Int}\left(\frac{\theta_p}{T}\right) + 1 \quad (12)$$

The remaining tuning parameters are related to the closed-loop speed of the response. Hence, the tuning guidelines are based on the closed-loop time constant of the system. In general, a small closed-loop time constant corresponds to a faster closed-loop response with large variations in the controller output. A large closed-loop time constant displays a slower and smoother response.

Tyreus and Luyben [18] showed that the closed-loop time constant for a FOPDT integrating model can be approximated as:

$$\tau_{CL} = \theta_p \sqrt{10} \quad (13)$$

The prediction horizon, P , and the model horizon, N , are computed as the closed-loop process settling time in samples as:

$$P = N = \text{Int}\left(\frac{5\tau_{CL}}{T}\right) + k \quad (14)$$

Note that both N and P cannot be selected independent of the sample time.

A larger P improves the nominal stability of the closed loop. For this reason, P is selected large enough such that it includes the steady state effect of all past controller output moves. N must be equally long to include the closed loop settling time of the process to avoid truncation error in the predicted process variable profile [14].

The control horizon, M , must be long enough such that the results of the control actions are clearly evident in the response of the measured process variable. The tuning rule thus chooses M as:

$$M = \text{Int}\left(\frac{\tau_{CL}}{T}\right) + k \quad (15)$$

The final step is the calculation of the move suppression coefficient, λ . Past researchers [10] have indicated that the move suppression coefficient, λ , suppresses aggressive controller action when $M > 1$, and improves the conditioning of the system matrix by rendering it more positive definite.

Previous work has shown that the choice of the move suppression coefficient for DMC can be made independent of the process gain [12, 13]. Gain-scaling represents a modification where a mathematical expression is stripped of the effects of the process gain for analysis independent of the gain.

For integrating processes, the move suppression coefficient must be gain and time-scaled since the units for the process gain are:

$$K_p^* [=] \frac{y(t)}{u(t) \cdot t} \quad (16)$$

The expression for the move suppression coefficient, λ , is given by Cooper and Dougherty [23]:

$$\lambda = \frac{1}{10M} \left(\frac{M^2(P-k+1)^3}{3} - 0.08M^3(P-k+1)^2 \right) [K_p^* T]^2 \quad (17)$$

The tuning guidelines for single input single output (SISO) DMC of self-regulating and non-self regulating processes are summarized in Table 1.

	Self Regulating	Non-Self Regulating (Integrating)
Step Response Model	$\frac{Y(s)}{U(s)} = \frac{K_p^* e^{-\theta_p s}}{\tau_p s + 1}$	$\frac{Y(s)}{U(s)} = \frac{K_p^* e^{-\theta_p s}}{s}$
Sample Time, T	$T = \text{Max}(0.1\tau_p, 0.5\theta_p)$	$T \leq 0.5\theta_p$
Discrete Dead Time	$k = \text{Int}\left(\frac{\theta_p}{T}\right) + 1$	$k = \text{Int}\left(\frac{\theta_p}{T} + 1\right)$
Closed Loop Time Constant, τ_{CL}	Not required	$\tau_{CL} = \theta_p \sqrt{10}$
Prediction Horizon, P Model Horizon, N	$P = N = \text{Int}\left(\frac{5\tau_p}{T}\right) + k$	$P = N = \text{Int}\left(\frac{5\tau_{CL}}{T}\right) + k$
Control Horizon, M	$M = \text{Int}\left(\frac{\tau_p}{T}\right) + k$	$M = \text{Int}\left(\frac{\tau_{CL}}{T}\right) + k$
Move suppression coefficient, λ :	$\lambda = \frac{M}{10} \left\{ \frac{3.5\tau_p}{T} + 2 - \frac{(M-1)}{2} \right\} K_p^2$	$\lambda = \frac{1}{10M} \left(\frac{M^2(P-k+1)^3}{3} - 0.08M^3(P-k+1)^2 \right) [K_p^* T]^2$

TABLE 1. TUNING GUIDELINES FOR SISO DMC

IMPLEMENTATION EXAMPLE 1

This example demonstrates the shortcomings of the original Smith predictor architecture and the benefits of the modified architecture when implemented on a pure integrator process. Consider the following system:

$$G_p(s) = \frac{0.2}{s} e^{-5s} \quad G_D(s) = 1 \quad G_C(s) = K_C \left[1 + \frac{1}{\tau_I s} \right] \quad G_0(s) = \frac{0.2}{s} \quad G_T(s) = e^{-5s} \quad F(s) = 0.5 \quad (18)$$

A disturbance step followed by a set point step is applied to the system. A classical PI controller and a PI with Smith predictor were tuned using the procedure outlined in [17, 24]; with the PI controller tunings for the Smith predictor calculated with no time delay.

As shown in Fig. 4, the original Smith predictor exhibits an offset equal to $K_p^* \theta_p$ in the presence of a type-I disturbance. The classical PI controller is able to return the system back to set point, although the response is sluggish. The modified Smith predictor shows a clear benefit over the PI controller and original Smith predictor with a smaller peak overshoot ratio and a faster settling time. Not shown is that

in the presence of a type-II disturbance, the Smith predictor requires no modification and functions as expected.

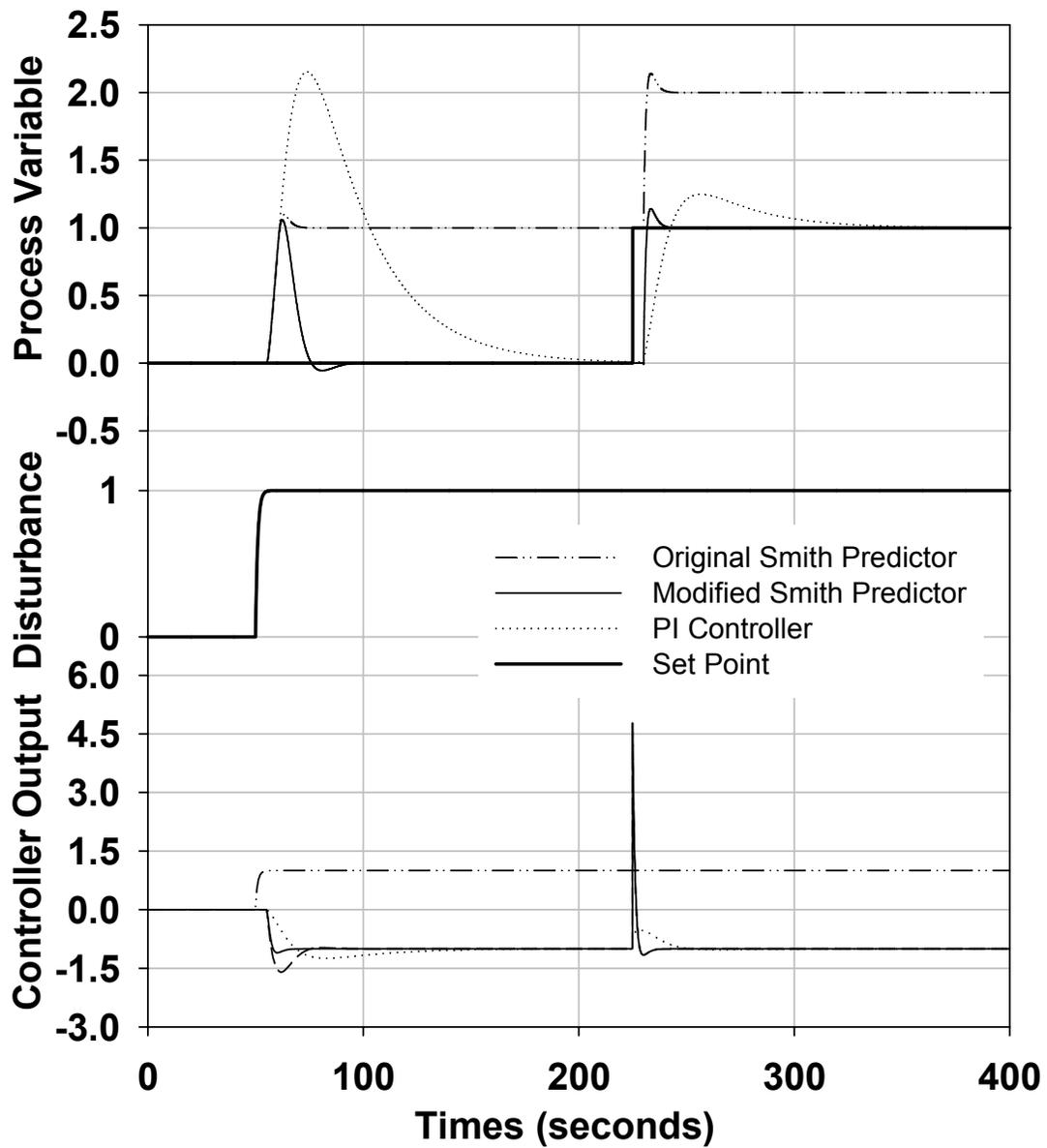


FIG. 4 - PERFORMANCE FOR A TYPE-I DISTURBANCE.

IMPLEMENTATION EXAMPLE 2

This example demonstrates the capability of the DMC controller designed with the FOPDT Integrating model. The transfer function of a viscosity loop in a polymerization process is specified as [4]:

$$G_p(s) = \frac{3.0e^{-10s}}{100s + 1} \quad (19)$$

The controller output of this loop is the set point of an inner pressure control loop in a cascade arrangement [4]. This process has an open-loop time constant of 100 minutes and a process dead time of 10 minutes. Previous researchers [17] have shown that an alternative approach to controller design is to model the initial dynamics as an integrating process. Figure 5 (left) shows the open loop response of Eq. (19) where the controller output is stepped from 50 to 55. The right plot in Fig. 5 shows the initial dynamics of Eq. (19), a FOPDT Integrating model is fit to the initial dynamics to yield Eq. (20).

$$G_M(s) = \frac{0.027e^{-9.16s}}{s} \quad (20)$$

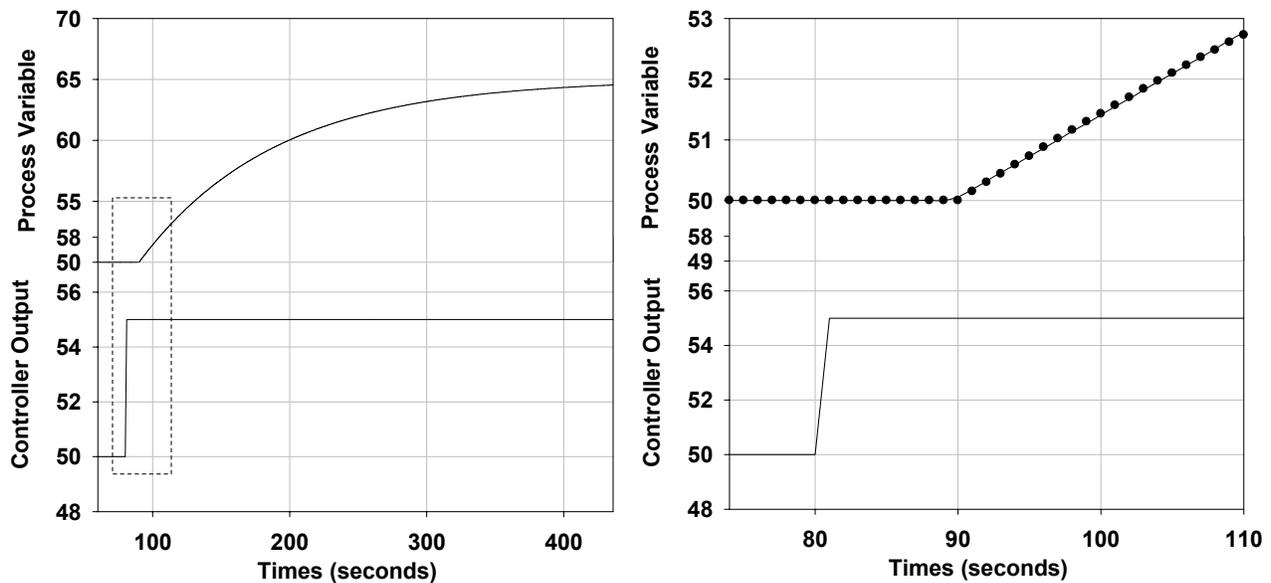


FIG. 5 - STEP RESPONSE OF (19) (LEFT). INITIAL DYNAMICS AS EQ. (20) (RIGHT).

Two demonstration runs are presented in Fig. 6. Equations (19) and (20) are used in conjunction with Table 1 to produce the DMC controller tunings shown in Table 2. Figure 6 shows the set point tracking and the disturbance rejection performance of DMC controller designed using the self regulating DMC tuning guidelines. The solid line shows the DMC controller designed using the non-self regulating tuning rules.

	Self Regulating	Non-self Regulating (Integrating)
Sample Time, T	10	4.58
Prediction Horizon, P; Model Horizon, N	52	34
Control Horizon, M	12	9
Move suppression coefficient, λ :	340.2	142.98

TABLE 2. DMC TUNING PARAMETERS

As illustrated by Fig. 6, both sets of tuning guidelines are able to give stable responses and similar set point tracking performance (left plot). The non-self regulating tuning guidelines provide for improved disturbance rejection in that they minimize the maximum deviation and shorten the settling time at the expense of increased controller activity (right plot).

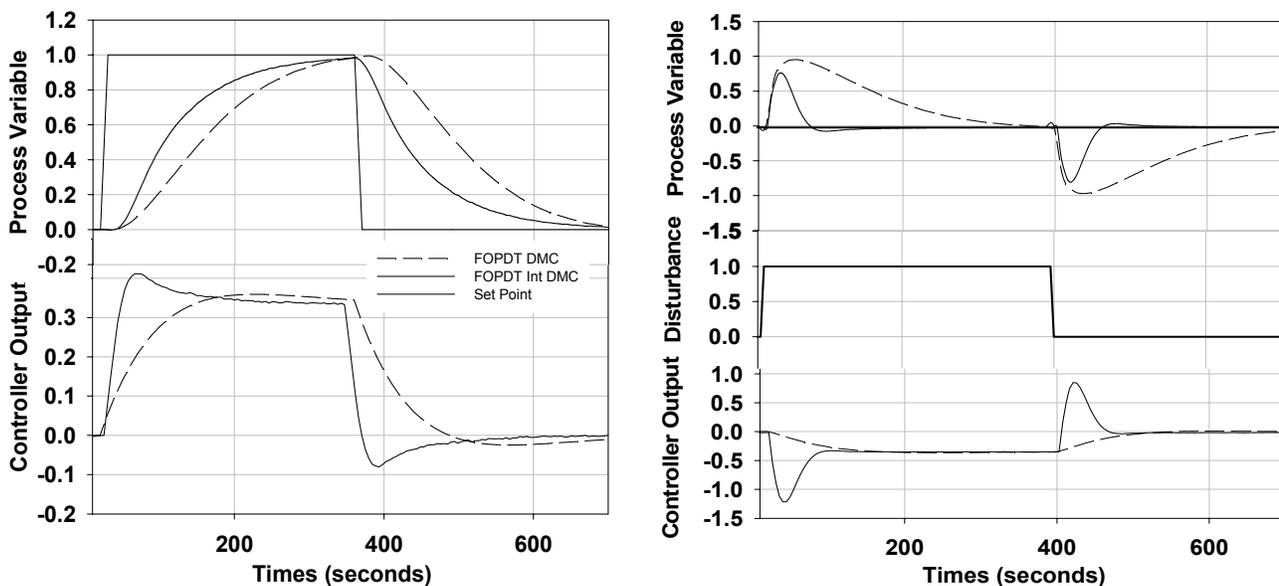


FIG. 6 - (LEFT) SETPOINT TRACKING. (RIGHT) DISTURBANCE REJECTION.

CONCLUSIONS

Methods for MPC design and tuning have been presented and demonstrated. The general method entails collecting dynamic process test data near the design level of operation, fitting an integrating dynamic model to this test data and using the resulting model parameters in correlations to compute controller tuning values. A first example shows that a modified Smith predictor provides a clear benefit over the original Smith predictor design in the presence of a type-I disturbance. Specifically, the modified Smith predictor eliminates the steady state offset that has been observed and studied by others. A second example shows that FOPDT Integrating DMC tuning guidelines provide a fast closed loop response in disturbance rejection scenarios when employed on a self regulating process. This improved performance comes at the cost of increased controller activity.

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