Constraints handling in multivariable system by managing MPC squared controller

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This paper considers how to achieve best dynamic response from a multivariable predictive controller. Condition number is used as a main criterion. High condition number reflects collinear dependence of some process inputs and outputs. Another criterion for building an optimal controller is controller performance, which can be best achieved with a squared controller configuration. These criteria lead to an optimal multivariable controller that utilizes a squared controller with minimal condition number. Constraint handling in such a controller is achieved by managing controller set points determined by an LP optimizer. To get faster controller response, which is important for handling constraints, in addition to the condition number, process dynamics too are taken into account in developing the controller configuration. These considerations are used in developing a procedure for industrial controller design. The procedure, along with preliminary test results is outlined in the paper. The tests present controller performance and constraints handling on a simulated distillation unit.

Keywords

Constrained Model Predictive Control, Dynamic Matrix Control, Matrix Condition Number, Penalty on Moves

Introduction

Designing MPC setup is one of the essential steps in developing an effective controller for a multivariable application [1]. The major criterion for defining a multivariable controller is controllability. Controllability can be defined for practical purposes as the ability to track set point changes and handle constraints by compensating for measured and unmeasured disturbances. Normally the process configuration is non-squared [2], and in general it has more outputs (controlled and constrained variables) than manipulated variables, but number of manipulated variables is greater than or equal to the number of control variables. Also, a control range around the set point may be defined for each control parameter. These extra numbers of manipulated variables and the control range create degrees of freedom that can be used for constraint handling and optimization. The number of active constraint variables changes dynamically during plant operation. To account for changing requirements, typically, MPC controller is reconfigured appropriately at each execution cycle [3]. However, controller reconfiguration changes its controllability and the MPC may become ill conditioned [3]. Ill conditioning happens when process response matrix used for MPC controller generation is collinear, such that two or more process outputs respond in nearly identical fashion to a change in the manipulated variable. The controller moves in such conditions are excessive and may be easily saturated, bringing controller operation to a halt. Ill conditioning can be removed by singular value thresholding [3], which involves decomposing the process model using singular value decomposition. Singular values below a threshold magnitude are discarded, and a process model with a much lower condition number is then reassembled and used for control. Other techniques use priorities or weights and drop the less important outputs from the MPC configuration in order to achieve better controllability. Finally, applying higher penalty on move it is possible to remove or minimize the effect of ill conditioning. However, this may be not acceptable for responsive control.

The latest generation of the MPC controller is embedded into scalable DCS controller, providing better reliability, easier application development and operation [4, 5]. Therefore, new strategies for dealing with ill conditioning that are suitable for embedded implementations are required.

The primary requirement is that MPC controller is not regenerated or reconfigured after downloading into the DCS controller. Therefore the optimal squared MPC controller should be developed prior to download. During operation, MPC controller is managed by the optimizer to satisfy the dynamic control and constraint requirements. Specifications of the technique, a procedure for developing MPC controller and its operational features are detailed in the rest of this paper.

Controllability assessment

For a process gain matrix, A condition number of the matrix $A^T A$ tests its controllability. Condition number for matrix A is defined as [6]:

$$Cond(A) = \|A\| \|A^{-1}\| \tag{1.1}$$

matrix $A^T A$ is applied to obtain positive condition numbers. To avoid matrix inversion, condition number is calculated by using singular value decomposition (SVD) [7]:

$$Cond(A) = \frac{\sigma_1}{\sigma_n} \tag{1.2}$$

σ_1 - is maximum singular value

σ_n - is minimum singular value

Smaller condition number means better controllability. There are no strict criteria for defining degree of controllability. Therefore, condition number can be used only as a relative comparison of various control matrices. Condition number for ill conditioned matrix may be in thousands and may approach infinity. Ill conditioning occurs in the case of collinear process variables – that is, due to collinear rows (or columns) in matrix A. In a way ill conditioning can be assessed as well by cross-correlation between matrix lines or rows.

It is apparent that proper selection of the input-output variables in the control matrix can reduce conditioning problems. The process gain matrix selected for controller is assumed to be square in the number of inputs and outputs. MPC application may have unequal number of MVs (manipulated variables) and CVs (control and constraint variables). Naturally, all the available MVs are included in the dynamic controller configuration. Hence, there are two aspects that need to be considered in controller generation:

- Selection of the process output (control or constraint) variables that are made part of the dynamic control matrix.
- Realizing a square process control matrix that is not ill conditioned. Resulting matrix is square in terms of number of available MVs.

Condition number is calculated from steady state gains and therefore defines controllability for a steady state. Accounting for process dynamics (dead time, lag) and model uncertainty has an effect on dynamic controllability. In addition, priority of process variables may dictate their inclusion in dynamic control. The focus here is in choosing the process output variables (control or constraint CVs) that are part of the square controller matrix.

It is possible to build a heuristic procedure intended to improve both steady state and dynamic controllability. The procedure would have several phases and may provide some improvements of the control matrix. Such procedure would have number of heuristic criteria, possibly some contradictory.

An example of concept implementation presented in the next sections considers both steady state and dynamic controllability. For steady state controllability a simple automatic procedure for building the default configuration is proposed, which takes into account primarily external user requirements and correlation between gain matrix lines, tested by successive condition number calculations. Optimizer target definitions, an interactive controller configuration dialog and heuristics account for the dynamic properties.

Handling Control and Constraint Variables by squared MPC controller

Handling constraints by the squared controller is a concept particularly suitable for the embedded MPC. Instead of reconfiguring controller for constraint handling as in traditional implementations, the squared controller repositions set points to drive the manipulated variables (and so in turn the process outputs) to the optimal steady state values determined by the optimizer. Optimizer solution accounts for constraints, process input limits and economic objectives.

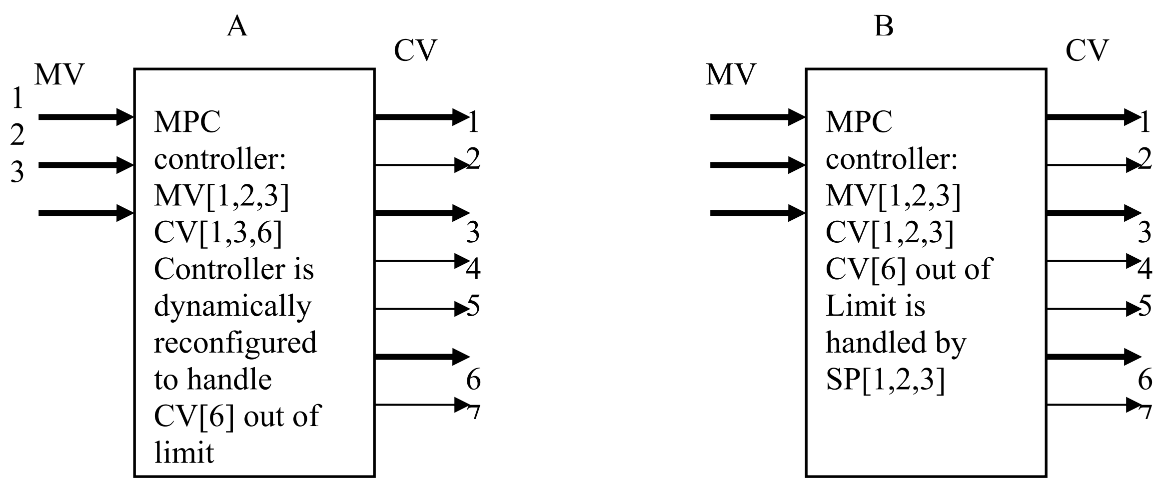


Figure 1. Handling constraints by reconfiguring MPC controller (A) or by managing MPC controller (B)

Figure 1 shows that one way to deal with constraints is to reconfigure controller to include outputs out of limits into MPC configuration (A). The approach in a sense emulates traditional PID override control. An alternative is to develop a responsive squared controller and handle constraints by managing this controller (B). The concept is supported by the following mathematics:

Predicted process output steady state equation for an *m* by *n* input-output process, with prediction horizon *p,* control horizon *c*, in the incremental form is:

$$\Delta CV(t+p) = A * \Delta MV(t+c) \tag{1.3}$$

where

$\Delta CV(t+p) = \left[\Delta cv_1, ..., \Delta cv_n\right]^{\mathrm{T}}$ denotes vector of the predicted changes in outputs at the end of prediction horizon,

$A = \begin{bmatrix} a_{11} ... \ a_{1m} \\ .... \\ a_{n1} \quad a_{nm} \end{bmatrix}$ the process steady state *m x n* gains matrix,

$\Delta MV(t+c) = \left[\Delta mv_1, ..., \Delta mv_m\right]^T$ denotes vector of changes in manipulating variables at the end of control horizon

Vector $\Delta MV(t+c)$ represents the sum of the changes over control horizon made by every controller output mv_i.

$$\Delta mv_i = \sum_{j=1}^{c} mv_i(t+j) \qquad i = 1, 2, ..., m$$

The changes should satisfy limits on both MVs and CVs.

$$MV_{\min} \le MV_{current} + \Delta MV\left(t+c\right) \le MV_{\max} \tag{1.4}$$

$$CV_{\min} \le CV_{predicted} + \Delta CV\left(t+p\right) = CV_{predicted} + A * \Delta MV(t+c) \ \le CV_{\max} \tag{1.5}$$

The optimizer solves an objective function such that the process outputs (CVs) and MVs stay within their constraints and limits. An economic objective function for maximizing product value and minimizing raw material cost can be defined jointly in the following way:

$$\underset{\min}{Q} = -UCV^T * \Delta CV(t+p) + UMV^T * \Delta MV(t+c) \tag{1.6}$$

UCV is cost vector for a unit change in CV process value;
and *UMV* is cost vector for a unit change in MV process value.

Applying (1.3), the objective function expressed in terms of MV only is:

$$\underset{\min}{Q} = -UCV^T * A * \Delta MV(t+c) + UMV^T * \Delta MV(t+c)$$

Optimal MV values are applied to the MPC control as the target MV values to be achieved within control horizon. If MPC controller is squared, i.e., the number of MVs is equal to the number of controlled variables, then MV targets can be effectively achieved by change in CV value.

$$\Delta CV^C = A^C * \Delta MVT \tag{1.7}$$

ΔMVT - optimal target change of MV to handle constraints and optimize objective function
A^C - subset of process model matrix used for MPC controller generation
CV^C - subset of control and constraint variables redefined as control variables and included into MPC squared controller, ΔCV^C - CV^C change to achieve optimal MV, implemented by managing set points of the output variables included in the square MPC controller.

Building MPC Controller with well defined controllability

To achieve fast changes of MV, the matrix A^C should be properly selected. The primary criterion for the selection is matrix condition number. The MPC controller is generated from the process model and controller design parameters. It minimizes the squared error of controlled variables over the prediction horizon and the squared error of controller output (manipulated variables) over the control horizon. This control objective function is given by:

$$\min_{\Delta \mathbf{U}(k)} \left\{ \left\| \mathbf{\Gamma}^y [\mathbf{X}(k) - \mathbf{R}(k)] \right\|^2 + \left\| \mathbf{\Gamma}^u \Delta \mathbf{U}(k) \right\|^2 \right\} \qquad (1.8)$$

where **X(**k**)** is the controlled output *p*-step ahead prediction vector, **R(**k**)** is the *p*-step ahead reference trajectory (set point) vector, $\Delta\mathbf{U}(k)$ is the *m*-step ahead incremental controller moves vector, $\mathbf{\Gamma}^y = \mathrm{diag}\{\Gamma^y_1, \ldots, \Gamma^y_p\}$ is a penalty matrix on the controlled output error, $\mathbf{\Gamma}^u = \mathrm{diag}\{\Gamma^u_1, \ldots, \Gamma^u_m\}$ is a penalty matrix on the control moves, *p* is the prediction horizon (number of steps) and *m* is the control horizon (number of steps).

The solution that minimizes this objective function as given in [8] is:

$$\Delta \mathbf{U}(k) = \left(\mathbf{S}^{u^T} \mathbf{\Gamma}^{y^T} \mathbf{\Gamma}^y \mathbf{S}^u + \mathbf{\Gamma}^{u^T} \mathbf{\Gamma}^u \right)^{-1} \mathbf{S}^{u^T} \mathbf{\Gamma}^{y^T} \mathbf{\Gamma}^y \mathbf{E}_p(k) \qquad (1.9)$$

where $\mathbf{S}^u$ is the process dynamic matrix built from the step responses: of dimension *p* x *m* for SISO model and *p*no* x *m*ni* for MIMO model with *ni* inputs and *no* outputs, $\mathbf{E}_p(k)$ is the error vector over the prediction horizon, and superscript of *T* denotes a transposed matrix.

As can be seen, matrix inversion is a key requirement. From a strictly mathematical standpoint, a square matrix provides well-defined behavior. From a process standpoint too, it is intuitive that optimal control action can be achieved if a) the most responsive dependent variables (control or constraint CVs) are included and b) the number of dependent variables is equal to the number of independent variables (MVs). In order to meet this objective, matrix A^C is square in terms of the number of available MVs and the most responsive CVs. An automatic procedure for this is:

1. Selecting all control variables defined as CVs with required set point control initializes the controller configuration. Priority, gain or phase response, user input may also determine which CVs are used. If the number is equal to the number of MVs, proceed to Step 3, else Step 2.
2. Add one constraint variable at a time to the already selected configuration and calculate the resulting condition number. This test is performed sequentially for each constraint variable not yet selected for the configuration. Select the constraint variable that results in minimum condition

number, thereby expanding the configuration by one output at a time. If matrix is now square, go to Step 3, else add another constraint variable.

3. Calculate condition number of the final matrix. If resulting condition number of the matrix A^C is still too high, proceed to Step 4, otherwise Step 6.
4. It is possible to improve the condition number by removing one or more output control or constraint variables and applying wrap around (or self-controlled) MV. The wrap around procedure is performed by placing, in turn, a unity response for each of the different MVs in place of the removed control or constraint CV. The response is unity at one position (Gain = 1.0, Dead Time = 0, Time Const. = 0) and zero elsewhere. For example, for a four by four matrix, the combinations 1000, 0100, 0010 and 0001 will be placed in the row of the removed output line in the gain matrix A^C. In operation, the optimizer will change this MV by applying the target set point to the MPC controller, rather than managing MV directly. The controller achieves this set point as it would a set point of the real CV, i.e., it moves MV toward this set point to minimize the effect on the real CVs. Essentially, the wrap around makes it possible to move MVs directly by the optimizer rather than just by the dynamic controller with the added feature that the controller move is smooth (no MV bump) since it has set point control. The wrap around MVs can be applied in MPC controller configuration with the optimizer as well for simple pusher optimization [4].
5. After wrap around, select the combination that results in the minimum condition number. If there is no improvement, keep the original matrix.
6. At this point, the final square matrix configuration is obtained. Now, based on the process response (gains, dynamics) associate every selected control or constraint CV with an MV, excluding the wrap around MV, if any. As a result of this pairing, each control or constraint parameter in the square control configuration is paired to a unique MV, and this is used in automatically calculating the penalty on moves (PM) for each MV. As is well known, the PM factors are the most important tuning factor for controller generation. The pairing delivers PM values that best satisfy the conflicting requirements of performance and robustness [9].

Note that since the matrix is multivariable (square in terms of MVs), the pairing maintains the relationships between the non-paired variables. Such a controller configuration is shown in Figure 3 for the Shell Heavy Oil Fractionator (HOF) problem [10] described in some detail in the next section. The full HOF is a non-square 5x7 process with 3 manipulated variables (MVs), 2 disturbance variables (DVs) and 7 output variables (CVs: 2 control and 5 constraint). The automatic configuration results in a square 3x3 control matrix with the calculated condition number as shown in Figure 3. For the selected MV (TOP_DRAW), the response parameters are shown in the left pane. User selection to modify the control matrix configuration is done via the add/delete buttons. The condition number of changed configuration is also shown in direct comparison with the automatically selected one. The main reason for modifying automatic configuration is to account for process dynamics (lags and dead times), to include variables deemed important, and to build a more responsive controller with acceptable condition number. The control generation routine (1.9) then uses the step responses of the variables included in the square control matrix to realize the MPC controller. As described above, the optimizer targets ensure that dynamic control operation meets the set points without violating constraints for all the parameters, including those not in the square controller.

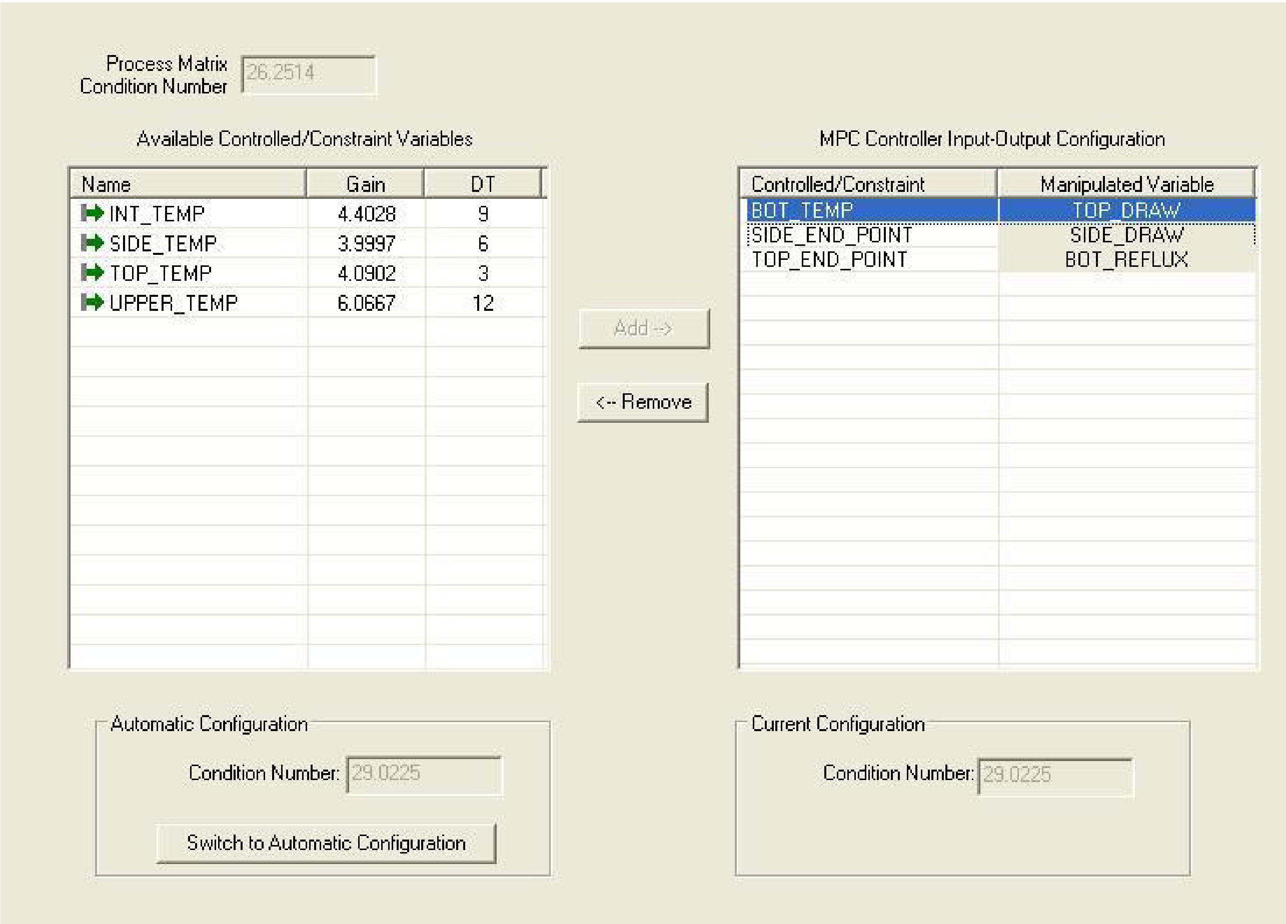


Figure 3. Dialog window for developing optimal MPC control matrix configuration

Implementation example

The Shell HOF (Figure 4) has top and side drains with product specifications determined by economics and operating constraints. A bottom draw has no product specification. Heat enters the HOF with the gaseous feed and heat is removed from the column to achieve the desired specification in three side-circulating loops. The top two side-circulating loops are heat integrated with other parts of the plant. Since operational conditions in other units vary, they can be a source of disturbances on any HOF. In all test cases, step disturbances in the duties of the two loops must be rejected by any proposed control strategy. Heat removal from the bottom side-circulating loop is regulated by adjusting steam production. There is an operational constraint on the temperature in the lower part of the column. The figure also shows the measurement on the inlet of the bottom side circulation loop. The HOF model, presented in 1987 by Prett and Morari along with control objectives and constraints [10], is used as a reference MPC test. The process is difficult to control as model matrix lines are close to being collinear. Control response of the square MPC controller generated by the automatic procedure described earlier was tested on this non-square HOF process.

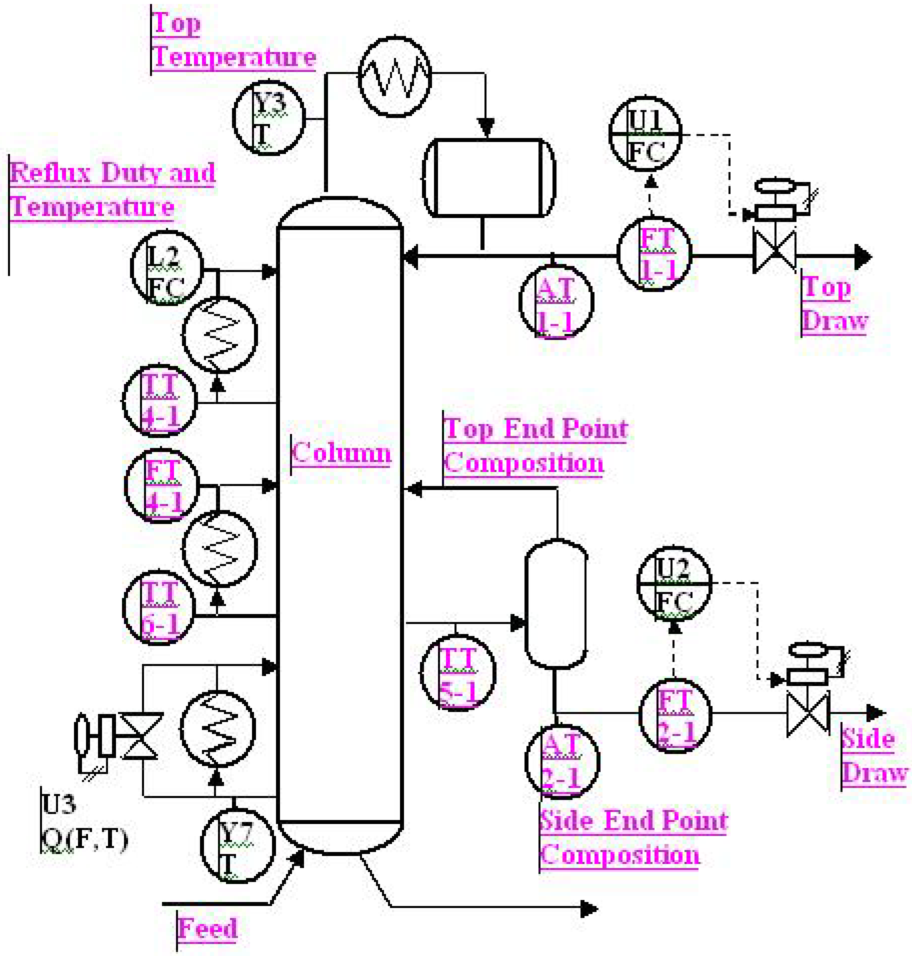
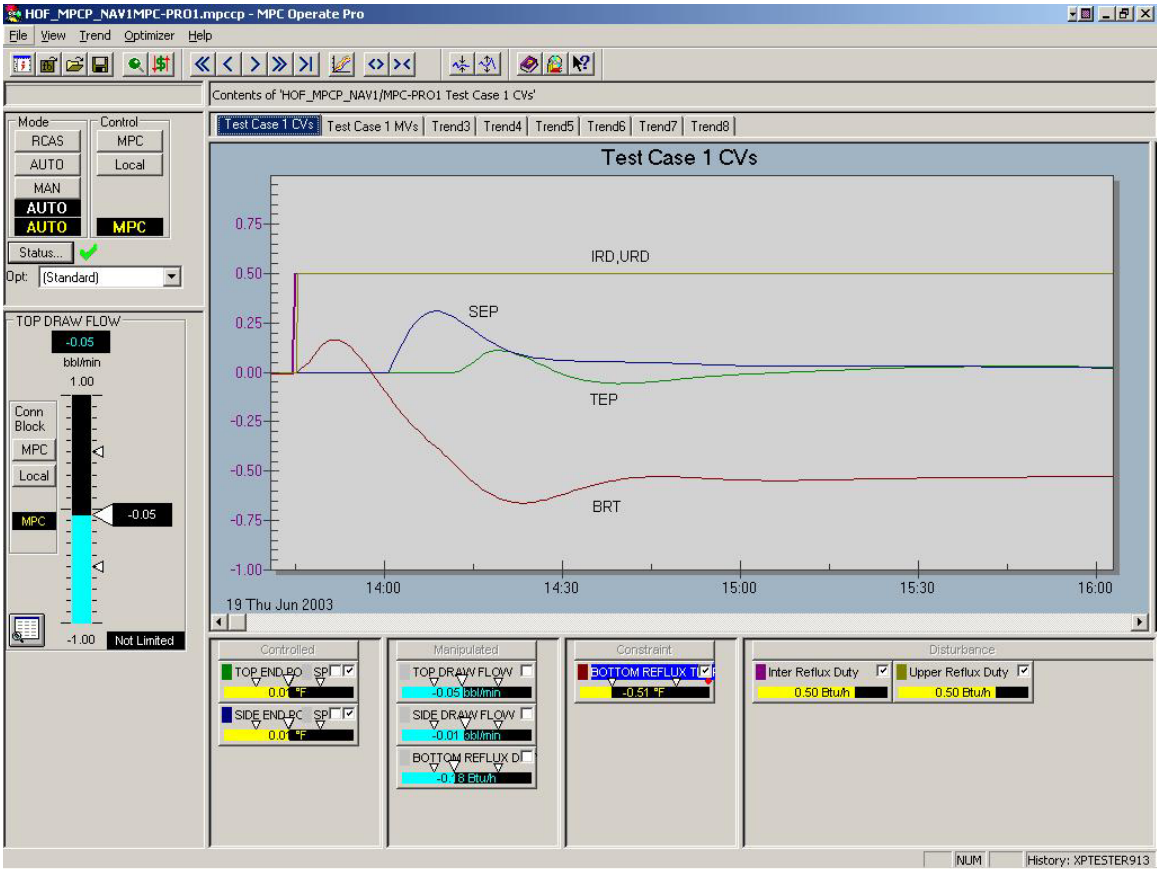


Figure 4. The Shell Heavy Oil Fractionator process

Figure 5 shows the MPC controller response to Test Case I (disturbance rejection to simultaneous step change, of magnitude 50% of limits, in both DVs) well within the required specification. The squared controller has been built by including the two control variables with set point control (TEP: Top End Point, and SEP: Side End Point) and one of the constraint variables (BRT: Bottom Reflux Temp), in order to match the three manipulated variables (TDF: Top Draw Flow, SDF: Side Draw Flow, and BRD: Bottom Reflux Duty). The two disturbances variables are IRD: Inter Reflux Duty, and URD: Upper Reflux Duty. In [11], the generated square MPC controller response to all five test cases of the Shell HOF problem, including problems with significant model mismatch, is shown to satisfy the required performance and robustness objectives.



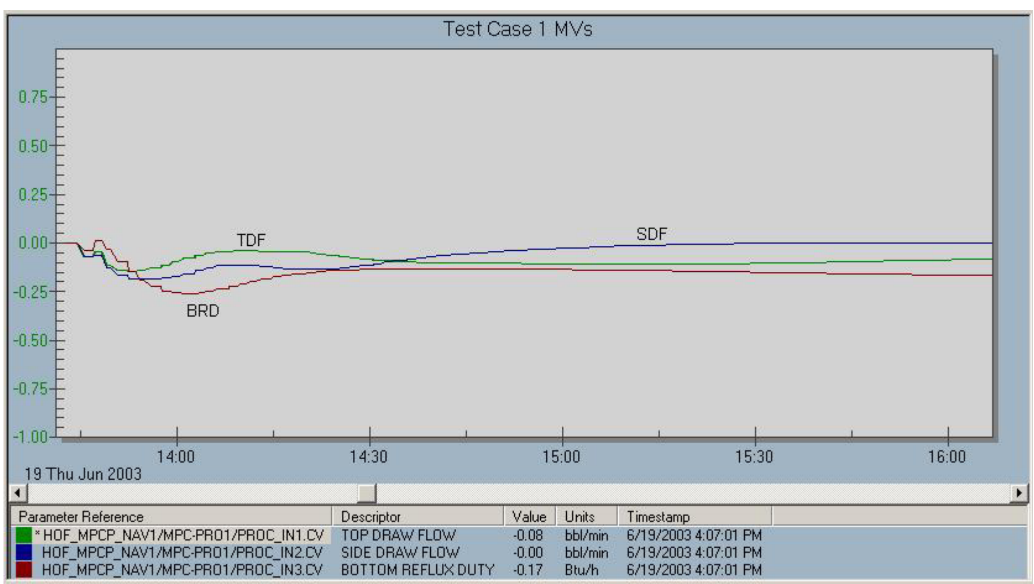


Figure 5. Test Case I response for the Shell model with square MPC controller.
Top: Square controller CVs (SEP, TEP, BRT) in response to step change in the DVs (IRD, URD). SEP and TEP stay within limits and achieve set points at steady state.
Bottom: Corresponding MV (TDF, SDF, BRD) moves remain within limits.

Conclusions

Developing a squared controller matrix with minimum condition number provides an optimal dynamic MPC configuration in respect of controllability. An automatic procedure is used to develop such a controller. Condition number can be further improved by applying wrap around manipulated variables. This, in turn with the optimizer in operation is a basis for good control and constraint handling for the total MPC (possibly non-square) configuration. The configuration can be manually adjusted based on dynamics. Test results simulating difficult to control models confirm the concept. In the future, the square matrix selection may be enhanced by using cross-correlation analysis between time series of the process variables and incorporating heuristics that pair variables in a more suitable fashion for dynamic control.

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